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Oscillations in double-quantum-well structures

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Abstract. The energy levels of a double-quantum-well system can be expected to occur in closely spaced pairs, due to 'tunnelling', so raising the possibility that oscillations can occur at the tunnelling frequencies. This paper gives a detailed theory of the excitation of such a pair by an incident electron wavepacket, using a time-dependent formalism and a one-dimensional system of barriers. It is confirmed that transient oscillations can be expected to occur in currents and charge densities near quasi-resonances. It is further reasoned that to increase the temporal coherence of an incident pulse, it is desirable to have inelastic scattering into a quasi-bound state.

1. Introduction

Since the pioneering work of Chang, Esaki and Tsu, [1, 2],which found that suitably designed low-dimensional barrier structures have regions where the differential conductivity is negative, there has been much experimental and theoretical work to investigate this phenomenon more thoroughly [3–5]. It has been shown that this feature can give rise to oscillatory behaviour, or bistability. The purpose of this paper is to discuss another source of instability in structures similar to those that show the negative resistivity characteristic, but which arises by an entirely different mechanism.

To set the scene for a detailed discussion it is convenient to begin with a single one-dimensional well with infinite barriers. The Schrödinger equation for such a system is usually solved only for the central region, and the eigenstates are discrete. However, the equation also has solutions for the regions to the left and right of the central region, and in these the eigenvalues form continua. If the infinite barriers are then reduced to finite barriers the new problem has a continuum of energy levels, and the discrete nature of the states that were in the central region seems to have disappeared. The situation is frequently described by introducing the concept of quasi-bound or quasi-resonant states, these being states that are regarded as mainly concentrated in the central region with the property that an electron placed in one of them will, in due course, leak out. Such a description needs justification, for eigenstates are stationary in time and this concept of a quasi-bound state would appear to be a time-dependent one. Nevertheless the idea is a useful one, and as will emerge, it can be placed on a firmer foundation.

The next step is to consider a system in which there are two wells, which for convenience are taken in the symmetrical form shown in figure 1. There would now seem to be two sets of quasi-bound states, each state being associated with both wells.

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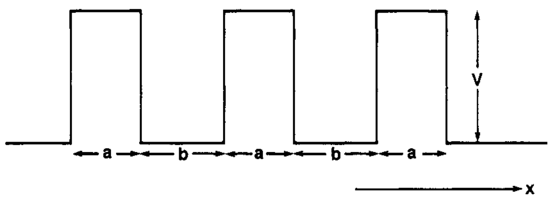

Figure 1. Diagram of the one-dimensional model of a symmetric double-well, triple barrier heterostructure used in this paper. The barrier height is V, the well width b and the barrier width a.

A number of experiments have shown that the pairs of quasi-bound states exist [6] (and they have previously been calculated theoretically [1]) in suitable layer systems composed of semiconductors. They are not however, at least *a priori*, described by potential barrier systems such as figure 1 would suggest, for they have more complicated properties, and the experiments usually induce carriers from bound states into quasi-bound states by absorption of electromagnetic waves [6–8]. These experiments do not, as far as is known, distinguish between the two wells for a wholly symmetric system since excitation only occurs between eigenstates [6,8,9]. In an asymmetric system, however, where eigenstates may be largely confined to specific wells, optical excitation may then distinguish between wells [10]. Luryi [4] has given an analysis of similar systems assuming that the eigenvalues are discrete. In fact they form a continuum, although some will have wavefunctions that are concentrated mainly in wells. It is our belief that under weak electromagnetic excitation the transitions take place between eigenstates. If structure is observed, this is because of the variation of selection rules with frequency and does not, *a priori*, indicate that the transitions which have been induced have a spatial significance. On the other hand, evidence of oscillatory behaviour may be obtained using intense and coherent electromagnetic radiation [11]. The passage of a current of electrons from the left to the right would always seem to distinguish between the two wells, for an electron must first tunnel into the left-hand well before it can tunnel into the right-hand one. There is then some reason to suppose that an oscillating current will be set up, with a frequency determined by the separation of the quasi-bound pair. The purpose of this paper is to give the theory of such an effect, in a simple example.

The method will use an extension of one previously used in several tunnelling problems [5], and amounts to the time-dependent solution of the Schrödinger equation with, as the initial condition, a pulse to the left of the barrier system. The barrier system will be about the simplest one which exhibits the phenomenon to be investigated, and its solution is of interest because it can be used to infer what may occur with more complicated barrier structures.

2. The time-independent solution

Using the barrier system of figure 1, the first step is to set up solutions of

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi$$

taking the solution for $x \leq 0$, and $V(x) = 0$, to be of the form

$$\mathrm{e}^{-\mathrm{i}wt}[\mathrm{e}^{\mathrm{i}kx} + R_0(k)\,\mathrm{e}^{-\mathrm{i}kx}]$$

where

$$w = \hbar k^2/2m\,.$$

That is, an incident plane wave of unit amplitude from the left and a reflected part with amplitude $R_0(k)$. The solution for all other values of x then follows. For regions where the potential is zero, the solutions take the form

$$\mathrm{e}^{-\mathrm{i}wt}[R_n(k)\,\mathrm{e}^{\mathrm{i}kx} + L_n(k)\,\mathrm{e}^{-\mathrm{i}kx}]$$

with complicated expressions $R_n(k)$ and $L_n(k)$ for the left and right travelling wave parts. For the moment it is not necessary to give these forms explicitly, though in due course it will become essential to study some of them in detail.

Similarly, within the barriers the solutions take the form

$$\mathrm{e}^{-\mathrm{i}wt}[A_n(K)\,\mathrm{e}^{Kx} + B_n(K)\,\mathrm{e}^{-Kx}]$$

where

$$\hbar w = -(\hbar^2 K^2)/2m + V\,.$$

3. The time-dependent solution

It will be assumed that at $t = 0$ the wavefunction is entirely to the left of the barrier system, where it is zero for $x < x_1$ and $x > x_2$, and $\exp(\mathrm{i}k_0 x)/\sqrt{L}$ for $x_1 \leq x \leq x_2 < 0$, with $L = x_2 - x_1$. Using Fourier transforms it can be written in the alternative form

$$\Psi(x,0) = \Psi_2(x,0) - \Psi_1(x,0)$$

where

$$\Psi_2(x,0) = -\frac{1}{2\pi\mathrm{i}\sqrt{L}} \int_{-\infty}^{\infty} \frac{1}{k - k_0} \exp[\mathrm{i}(k_0 - k)x_2]\,\mathrm{e}^{\mathrm{i}kx}\,\mathrm{d}k$$

and $\Psi_1(x,0)$ has x_1 in place of x_2.

For $t > 0$ it might appear that the time-dependent solution is

$$\Psi(x,t) = \Psi_2(x,t) - \Psi_1(x,t)$$

with

$$\Psi_2(x,t) = -\frac{1}{2\pi\mathrm{i}\sqrt{L}} \int_{-\infty}^{\infty} \frac{1}{k - k_0} \exp[\mathrm{i}(k_0 - k)x_2] \exp[\mathrm{i}(kx - wt)]\,\mathrm{d}k$$

which satisfies the Schrödinger equation for the region to the left of the barriers. It does not however satisfy the equation within any barrier, nor does it contain reflected components. The simplest procedure is to infer the true solution as follows. For $x < 0$ set

$$\Psi(x,t) = \Psi_2(x,t) - \Psi_1(x,t)$$

with

$$\Psi_2(x,t) = -\frac{1}{2\pi\mathrm{i}\sqrt{L}}\int \frac{1}{k-k_0}\{\exp[\mathrm{i}(k_0-k)x_2]\,\mathrm{e}^{-\mathrm{i}\omega t}[\mathrm{e}^{\mathrm{i}kx} + R_0(k)\,\mathrm{e}^{-\mathrm{i}kx}]\}\,\mathrm{d}k$$

and a similar expression for $\Psi_1(x,t)$ with x_1 replacing x_2. For x within a well set

$$\Psi_2(x,t) = -\frac{1}{2\pi\mathrm{i}\sqrt{L}}\int \frac{1}{k-k_0}\{\exp[\mathrm{i}(k_0-k)x_2]\,\mathrm{e}^{-\mathrm{i}\omega t}[R_n(k)\,\mathrm{e}^{\mathrm{i}kx} + L_n(k)\,\mathrm{e}^{-\mathrm{i}kx}]\}\,\mathrm{d}k$$

where $R_n(k)$ and $L_n(k)$ are the appropriate amplitudes for that well, and

$$\Psi_2(x,t) = -\frac{1}{2\pi\mathrm{i}\sqrt{L}}\int \frac{1}{k-k_0}\{\exp[\mathrm{i}(k_0-k)x_2]\,\mathrm{e}^{-\mathrm{i}\omega t}[A_n(K)\,\mathrm{e}^{Kx} + B_n(K)\,\mathrm{e}^{-Kx}]\}\,\mathrm{d}k$$

for x within a barrier. Then $\Psi(x,t) = \Psi_2(x,t) - \Psi_1(x,t)$ satisfies the Schrödinger equation and all matching conditions for all x, provided that the limits, which have been omitted in the above formulae, are taken to be the same for all integrals. The choice of limits is not arbitrary. For example, choosing the range from $-\infty$ to $+\infty$ for k does not obviously ensure that, at $t = 0$, there is no reflected component for $x < 0$ and no part already within the system of barriers. Any difficulty is removed by regarding k as a complex variable and choosing a common contour of integration for all the integrals that runs from $k = -\infty$ to $+\infty$ and that loops over the pole at $k = k_0$ and any others that are introduced by the R_n, L_n, A_n and B_n coefficients. The contours can then be deformed into the upper large semicircle without crossing any poles, when those integrals which should vanish are found to do so, leaving $\Psi(x,0)$ as simply the incident pulse. In this connection it is relevant to notice that, for a given incident k, the R_n, L_n, ... coefficients are determined solely by the structure of the barrier system, so their properties can be described as geometric. On the other hand the nature of the incident pulse shows itself in the $\exp[\mathrm{i}(k_0-k)x_2]/(k-k_0)$ and $\exp[\mathrm{i}(k_0-k)x_1]/(k-k_0)$ factors in the integrands, so these factors contain all the dynamical information. It is also useful to note that the Ψ_1 and Ψ_2 parts of Ψ can each be regarded as describing a semi-infinite pulse, one being displaced from the other. It follows that the overall behaviour can be obtained from the study of one of them, followed by displacement and subtraction.

4. Properties of the solution

For $t > 0$ it is not possible, in general, to evaluate the $\Psi(x,t)$ explicitly, and the most convenient approximation method is that of steepest descents, which is outlined in the appendix of this paper. The contours, which are initially identical, are then deformed as appropriate to the different regions of the barrier structure. For present purposes it is not necessary to examine the deformations in detail, for a good deal of insight can be obtained by asking how a 'quasi-resonance' can occur? In any chosen region of the barrier structure the integrand will have a pole at $k = k_0$ and, in general, some others due to whichever $R_n(k)$, $L_n(k)$, ... is involved. (As will appear they actually all have the same pole structure.) The contour of steepest descent, for a given choice of x, will change with time t, and as t increases it invariably happens that one or more poles will be crossed. As each pole is crossed the solution gains a residue, and the frequency which goes with this residue is simply obtained by taking the value of w appropriate to that value of k. That is, a pole at $k = k_1$ gives a residue with a factor $\exp(-\mathrm{i}w_1 t)$, where $w_1 = \hbar k_1^2/2m$. The pole does not in general lie on the real axis, and so the residue shows a damping with time. (It may be noted that within a barrier the pole is often found from an expression in K. A pole at $K = K_1$ gives $\hbar w = (-\hbar^2 K_1^2/2m)+V$, and since the corresponding k satisfies $(\hbar^2/2m)(k_1^2+K_1^2) = V$ it follows that again $w = (\hbar k_1^2)/2m)$. Since all the poles (except that at $k = k_0$) are of geometrical character, it can be anticipated that these residues describe the excitations of the natural resonances of the system as the pulse propagates through it. A detailed examination confirms these deductions, and it also demonstrates that, in general, these 'resonances' are only weakly excited. This too could have been anticipated, for the pulse is crossing barriers, so exponential damping can be expected.

The position changes drastically if a coefficient $R_n(k)$, and therefore each coefficient, has a pole which is close to k_0. Near these poles the integrand can be written in the form

$$\frac{C\exp[\mathrm{i}(kx - wt)]}{(k-k_0)(k-k_1)}$$

or as

$$\frac{C}{(k_0-k_1)}\left(\frac{1}{(k-k_0)} - \frac{1}{(k-k_1)}\right)\exp[\mathrm{i}(kx - wt)]$$

where C is slowly varying with k. When both poles contribute the total is, approximately

$$\frac{C}{(k_0-k_1)}\left(\exp[\mathrm{i}(k_0x - w_0t)] - \exp[\mathrm{i}(k_1x - w_1t)]\right)$$

and apart from any damping, the amplitude is enhanced by the $(k_0 - k_1)$ factor in the denominator. This illustrates the meaning of 'quasi-resonance', for the sum of the residues then consists of one damped and one undamped oscillation, both of large amplitude and small frequency difference.

In the two-well structure of figure 1 it can be expected that the poles of the coefficients, the 'quasi-resonances', will be in closely spaced pairs, so there is interest in what happens when k_0 is near a particular pair. Once the line of steepest descents has passed all three poles, an expression such as

$$\frac{\exp[\mathrm{i}(kx - wt)]}{(k-k_0)(k-k_1)(k-k_2)}$$

gives a pole contribution

$$\frac{\exp[\mathrm{i}(k_0x - w_0t)]}{(k_0-k_1)(k_0-k_2)} + \frac{\exp[\mathrm{i}(k_1x - w_1t)]}{(k_1-k_0)(k_1-k_2)} + \frac{\exp[\mathrm{i}(k_2x - w_2t)]}{(k_2-k_0)(k_2-k_1)} \cdots \tag{1}$$

which is a superposition of three time-dependent forms, each of large amplitude, and with small frequency differences. In particular, the square modulus, which determines

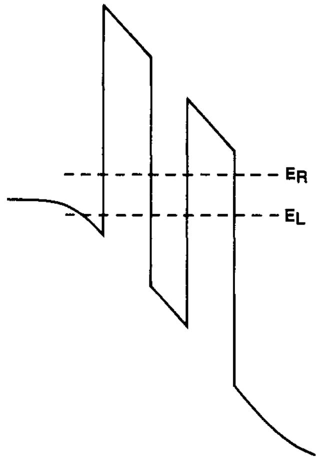

Figure 2. Diagram showing the probable potential profile of a double-barrier, single-well heterostructure. Here the system is shown off-resonance, such that E_l and E_r are far apart, and their associated wavefunctions Ψ_l and Ψ_r have peaks in their probability densities in different wells. As the voltage across the system is changed, the separation of the energy levels E_l and E_r is reduced until the levels anti-cross, at which point the states are described by the new wavefunctions ψ_1 and ψ_2, which are linear combinations of the old ones Ψ_l and Ψ_r.

the charge density, has a component at the frequency difference of the two quasi-resonances. This is, basically, the effect we wished to demonstrate, that excitation near a double quasi-resonance can be expected to give rise to charge and current oscillations at their difference frequency.

Strictly speaking, in a real system, this effect could be seen in a double-barrier resonant tunnelling structure, as shown in figure 2, since due to the formation of the accumulation layer, there are effectively two wells in the system. Initially the eigenvalues, which can be labelled E_l and E_r, are far apart and are associated with wavefunctions Ψ_l and Ψ_r which are centred in different wells. As the resonance condition is approached, the eigenfunctions change to forms such as

$$\frac{1}{\sqrt{2}}(\Psi_l \pm \Psi r)$$

The associated eigenvalues will not coincide, but will have a tunnelling separation, and it is this which gives rise to charge and current oscillations. As the oscillation frequency is determined solely by the geometry of the resonant structure, the possibility is that it can be tailored to obtain any chosen frequency. This conclusion has been reached on fairly general arguments and so can be expected with more complicated structures than are illustrated in figures 1 and 2. Nevertheless there is something to be said for illustrating it in more detail, so the following analysis will show how these properties arise for the structure of figure 1.

5. Detailed analysis

A convenient way to deal with a multi-well system is to consider figure 1, and set

$$\Psi(x,0) = r_n \exp[-\mathrm{i}k(a+b)]\,\mathrm{e}^{\mathrm{i}kx} + l_n \exp[\mathrm{i}k(a+b)]\,\mathrm{e}^{-\mathrm{i}kx}$$

so that the previously defined $R_n(k)$ and $L_n(k)$ satisfy

$$R_n(k) = \exp[-\mathrm{i}k(a+b)]\, r_n + \exp[\mathrm{i}k(a+b)]\, l_n$$

Then

$$\begin{bmatrix} r_{n+1} \\ l_{n+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} r_n \\ l_n \end{bmatrix}$$

where the elements a_{ij} are independent of n and define a transmission matrix with

$$\begin{aligned} a_{11} &= \frac{1}{2\mathrm{i}\sin 2\theta}\, \mathrm{e}^{\mathrm{i}kb}(\mathrm{e}^{Ka}\mathrm{e}^{2\mathrm{i}\theta} - \mathrm{e}^{-Ka}\mathrm{e}^{-2\mathrm{i}\theta}) \\ a_{22} &= \frac{1}{2\mathrm{i}\sin 2\theta}\, \mathrm{e}^{-\mathrm{i}kb}(-\mathrm{e}^{Ka}\mathrm{e}^{-2\mathrm{i}\theta} + \mathrm{e}^{-Ka}\mathrm{e}^{2\mathrm{i}\theta}) \\ a_{12} &= \frac{1}{2\mathrm{i}\sin 2\theta}\, \mathrm{e}^{\mathrm{i}kb}(\mathrm{e}^{Ka} - \mathrm{e}^{-Ka}) \\ a_{21} &= \frac{1}{2\mathrm{i}\sin 2\theta}\, \mathrm{e}^{-\mathrm{i}kb}(-\mathrm{e}^{Ka} + \mathrm{e}^{-Ka}) \end{aligned}$$

where $\tan\theta = k/K$, with $V = (\hbar^2/2m)(k^2 + K^2)$. For a sequence of N barriers, all that is necessary is to multiply the various transmission matrices together. For a pulse coming from the left the final r_N is taken to be zero. In the present case of two wells the matrix must be cubed. It is then apparent that every r_n (l_n) is a multiple of r_3, showing that they have the same pole structure as r_3. It's denominator is found to be

$$a_{21}a_{12}(a_{11} + a_{22}) + a_{22}(a_{21}a_{12} + a_{22}a_{22})$$

or

$$\begin{aligned} \mathrm{e}^{3Ka}\{&-\exp[\mathrm{i}(kb+2\theta)] + 2\exp[-\mathrm{i}(kb+2\theta)] - 3\exp[-3\mathrm{i}(kb+2\theta)]\} \\ &+ \mathrm{e}^{Ka}\{\mathrm{e}^{\mathrm{i}kb}(2\mathrm{e}^{2\mathrm{i}\theta} + \mathrm{e}^{-2\mathrm{i}\theta}) + \mathrm{e}^{-\mathrm{i}kb}(-2\mathrm{e}^{2\mathrm{i}\theta} - 4\mathrm{e}^{-2\mathrm{i}\theta}) \\ &+ 3\exp[-\mathrm{i}(3kb+2\theta)\} + \ldots \end{aligned} \qquad (2)$$

where the ... part has not been given specifically as it is not needed. It is readily determined by noting that the whole expression is reversed in sign if both K and θ are reversed. The quasi-resonances are then associated with the value of k which make this expression vanish. They are in general complex numbers.

To have a pole near k_0, which is real, implies that the zeros must be almost real, and since e^{Ka} is expected to be large, it would seem that a first approximation to the poles is obtained by setting the the terms in e^{3Ka} to zero. That is

$$\exp[\mathrm{i}(kb + 2\theta)] = \pm 1\,.$$

The next approximation is to set

$$k = k_0 + \Delta k \qquad \theta = \theta_0 + \Delta\theta \qquad K = K_0 + \Delta K$$

with $\exp[\mathrm{i}(k_0 b + 2\theta_0)] = 1$ and to include the terms of (2) in e^{Ka}. It is then found that for each choice of k_0 there are two solutions of Δk, which have real parts u of opposite signs, and imaginary parts v which have the same sign, where

$$u = \pm(\mathrm{e}^{-Ka}\sin 2\theta)/b \qquad v = -(\mathrm{e}^{-2Ka}\sin^2 \tfrac{1}{2}\theta)/b$$

$$\theta = n\pi/b\sqrt{U} \qquad U = 2mV/\hbar^2 .$$

It therefore follows that the separation in angular frequency, Ω_0, of the geometrical resonances is determined by $|2\hbar k_0 u/m|$ and so involves e^{-Ka}, whereas the widths of the resonances are determined by $|\hbar k_0 v/m|$ and so contain e^{-2Ka}. Inspection of the expression for u shows that the separation of the pairs of modes increases with their associated energies.

6. Numerical analysis

Because of the wide interest in resonant tunnelling, and our suggestion that oscillations will be found (which are analogous to those associated with the inversion splitting of NH_3), it is of interest to make some numerical estimates. Some results for the first (lowest energy) double quasi-resonance of a double-quantum-well structure (such as the one shown in figure 1) are shown in figures 3 and 4.

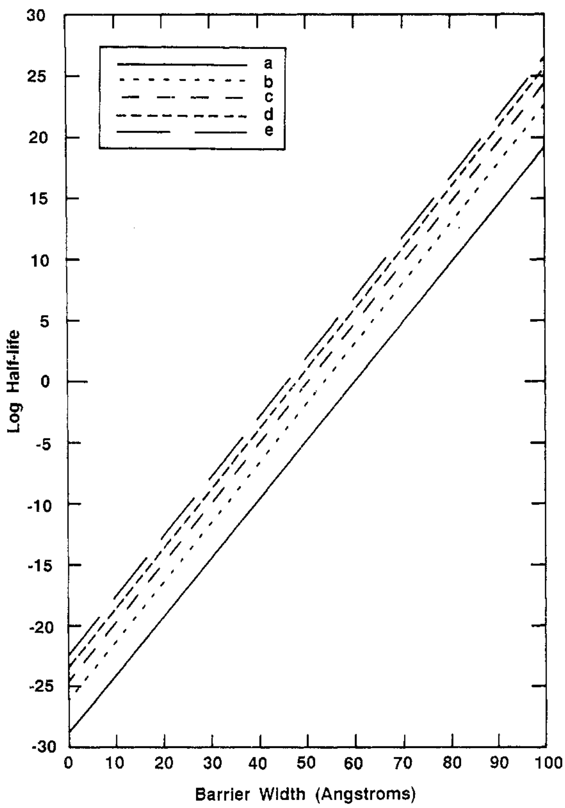

Figure 3. Plot of the logarithm of the half-life τ, in seconds, associated with excitation of the first double resonancce as a function of barrier width (in Å) for a symmetric triple barrier, double-well heterostructure for five different well widths (a), (b), (c), (d) and (e). Here (a) is 50 Å, (b) is 100 Å, (c) is 150 Å, (d) is 200 Å and (e) is 250 Å. The calculation was performed for the structure shown in figure 1 and uses a barrier height V of 230 meV, with an electronic mass of m_e, the rest mass.

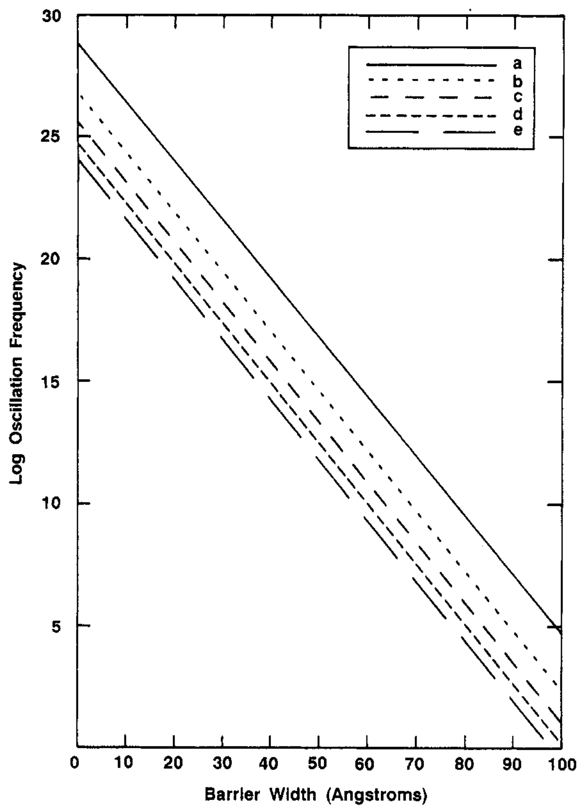

Figure 4. Plot of the logarithm of the oscillation frequency Ω_0, in Hz, associated with the first double resonance as a function of barrier width (in Å) for a symmetric triple barrier, double-well heterostructure for five different well widths (a), (b), (c), (d) and (e). Here (a) is 50 Å, (b) is 100 Å, (c) is 150 Å, (d) is 200 Å and (e) is 250 Å. The calculation was performed for the structure shown in figure 1 and uses a barrier height V of 230 meV, with an electronic mass of m_e, the rest mass.

7. Discussion and conclusions

The theory has used a particular shape of pulse incident on a barrier system, a shape which has been chosen so that any other shape can, by Fourier analysis, be expressed as a superposition of such pulses. The barrier region is taken to consist of potential barriers which are piecewise constant. This covers a wide variety of possibilities, but not all. Several features have emerged. In any given region the solution may be regarded as consisting of two parts, one of which is associated with a line integral and which arises from the method of steepest descents, and a second which arises from poles that may have been crossed by the line of steepest descents. If any such poles are isolated the contribution from the line integral, when the line goes near the pole, while small, is possibly greater than that from the pole. The position is however drastically altered if such a pole is not isolated, which is the special feature associated with resonant tunnelling. Then the pole contribution is likely to outweigh the line contribution. Indeed it is only under such circumstances that the tunnelling should be described as resonant, a characteristic which is associated experimentally with relatively large current flow through a barrier system. (To have current through a device a voltage must be applied across it: it is then incorrect to assume that the potentials remain piecewise constant. Nevertheless such barrier systems can be expected to have poles associated with quasi-resonances, and it is near coincidences with these poles which are, theoretically, needed for large current flows).

In addition the study has shown that particularly with double-well systems, it is possible to have two close geometrical resonances, which both enhance the pole contributions and produce current oscillations at their difference frequency. A similar effect was noticed previously in a many-electron system [12], which may be particularly relevant in the present context for, in an actual device, a description in terms of potential barriers is a tenuous concept, for really a many-electron treatment is needed. The inversion spectrum of NH_3 has already been mentioned, and there appears to be no obvious reason why inversion type oscillations should not occur in any system which has two or more closely spaced resonances, and can be driven by a current.

It is also of interest to note that not only does a potential profile such as that shown in figure 2 provide the conditions necessary for an oscillation in the current, but that the associated theory helps to resolve a long-standing problem in resonant tunnelling. Many papers use an analysis based on the eigenstates of a barrier system, these eigenstates being regarded as describing a flux of electrons. We prefer to regard the current as arising from a sequence of orthonormal wavepackets coming from the left, and accelerated by an applied electric field. As such packets will have come from a source it can be expected that their time duration will be determined by the properties of the source. It is to be expected that, in general, this time will be less than the half-life associated with the resonance in a barrier system, as it is determined in the main by inelastic scattering in the source, which is expected to be of order 10^{-12} s or less. If this is so then there will be little build-up of intensity within the barrier system due to multiple reflections, and little current will be transmitted. However, for a potential profile such as that shown in figure 2 the situation can be quite different. In this case an incident pulse may undergo inelastic scattering from the Fermi sea into the 2DEG situated in the accumulation layer next to the emitter barrier. As a result of this scattering the coherence time of the packet may be appreciably altered. If the time is lengthened it can be expected that the current through the system will be increased, due to the increase in intensity in the well. An examination of some experimental results supports this possibility. For a GaAs/AlGaAs symmetric quantum-well system [13] it has been found that the current through the first quasi-resonant state corresponds to an electron flux of $\simeq 10^{25}$ electrons s^{-1} m^{-2}. This comes from a charge density in the 2DEG [14] of $\simeq 10^{15}$ electrons m^{-2}, giving an estimated lifetime of $\simeq 10^{-11}$ s, a figure which seems to be too long to be associated with a source. To simulate the experimental conditions we have taken barrier widths as 56 Å, the barrier height as 32 meV., an effective barrier mass as 0.1 m_e, the well widths as 50 Å and the well effective mass as $0.067m_e$. This does not entirely simulate the structure of figure 2, for it is similar to the structure of figure 1. Nevertheless it produces a value for τ of $\simeq 10^{-11}$ s, which is of the same order as the observed value. We believe this confirms our suggestion that to increase the current through a barrier system it is necessary to have inelastic scattering into a trap before the barrier system, and to have this trap part of the quasi-resonance.

Acknowledgments

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Appendix. The method of steepest descents

The technique used in this paper involves separating an integral, $\Psi(x,t)$, into two parts; $\Psi_1(x,t)$ and $\Psi_2(x,t)$, and then treating them separately. $\Psi(x,t)$ alone has no pole at $k = k_0$, whereas $\Psi_1(x,t)$ and $\Psi_2(x,t)$ both have poles at $k = k_0$. Some care is needed in treating them near $k = \pm\infty$, and it is probably best to let the integrals run from $k = -R$ to $+R$, and then to let $R \to \infty$ at the end. The integrands tend to zero as $R \to \infty$.

The method of steepest descents is usually used for contour integrals of the type

$$\int_c \mathrm{e}^{\phi(z)}\,\mathrm{d}z \ldots \tag{A1}$$

where z is a complex variable, and c is a line between two points. At any z, $\phi(z)$ can be written as $\alpha + \mathrm{i}\beta$ where α and β are real. The problem to be circumvented arises because $\mathrm{e}^{\mathrm{i}\beta}$ will usually give an oscillating behaviour to the integrand, so a first step is to deform the contour of integration so that most of the integral is obtained by integrating along a line on which β is constant, then $\mathrm{e}^{\mathrm{i}\beta}$ can be taken outside the integral. The best line to follow is that which passes through a saddle point of $\phi(z)$, particularly if e^{α} is a rapidly decreasing quantity, for then the integral can readily be estimated. (The method is unsuitable if e^{α} is rapidly increasing). In general, e^{α} will decrease to zero, through positive values. There is however no guarantee that it will pass through the end points of c, so further line integrals (generally parts of the great circle) will be needed. However, in the cases of present interest, the integrands are zero at the end points, so it can be expected that the contributions from any such line integrals will be negligible.

In fact the integrals of interest are not of the form (A1), and as a next step it is convenient to study

$$\int_c \frac{\mathrm{e}^{\phi(z)}}{z - z_0}\,\mathrm{d}z\,.$$

This has a pole at $z = z_0$, and it may not be possible to follow the line of steepest descents, determined by $\mathrm{e}^{\phi(z)}$ without crossing the pole. In this case the integrand has an extra contribution of $2\pi\mathrm{i}\,\mathrm{e}^{\phi(z_0)}$. It is now obvious that if the integral has the form

$$\int_c \frac{F(z)\,\mathrm{e}^{\phi(z)}}{z - z_0}\,\mathrm{d}z\,.$$

where $F(z)$ has poles, some of these may also contribute.

In the above it has been convenient to take z as the complex variable. In the body of the paper the complex variable is usually k, but it can also be K. Also the expressions in which they appear usually contain two variables; x for position and t for time. The contours of steepest descent vary with x and t and a given pole contribution may or may not enter depending upon their values.

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